# Topic No. 2

## The Geostationary Orbit

We learned in ch. 2 that the orbital period is proportional to the semimajor axis. This is from kepler's third law.

$$a^3 \propto T^2$$

Thus for a satellite to have an orbital period of 24 hours, the semimajor axis must be of a specific value according to Kepler's Law. Such an orbit is called "Geosynchronous" orbit. That is a satellite makes one complete revolution in the same time the Earth makes one complete revolution around its axis.

However, this condition by itself is not sufficient to make the satellite appear stationary from Earth. What makes the orbit "Geostationary" are the following three conditions.

1- The orbit must be circular ($e \approx 0$)

2- The inclination must be zero.

3- The direction of travel must be from west to east (prograde orbit).

If an orbit has those conditions satisfied and its major axis is given by

$$a^3 = \frac{\mu}{\left(\frac{2\pi}{T}\right)^2} \Rightarrow a = \left(\frac{\mu T^2}{4\pi^2}\right)^{1/3}$$

$$T = 24 \text{ hrs} \times 60 \times 60 \quad \text{sec}$$

$$\Rightarrow a = a_{GSO} = 42{,}164 \text{ km}$$

Then the orbit is geostationary

There is only one orbit in space which satisfies all those conditions. Thus this orbit is considered a natural resource and its use must be regulated and divided among all nations (fairly).

The perimeter (length) of this orbit can be calculated roughly as :

$$L_{GSO} = 2\pi a_{GSO}$$

$$\simeq 265{,}000 \text{ km}$$

The height of the GSO from the Earth's surface at the equator is given by :

$$h_{GSO} = a_{GSO} - R_E$$

$$\simeq 35{,}790 \text{ Km}$$

[always remembered as 36,000 km]

## 3-1. Antenna look angles:

A satellite in the GSO appears fixed in the sky with respect to points on Earth. This is very useful for direct communication as the receiving antenna can be fixed looking in the direction of the satellite without any need for tracking.

It is usually required to determine three unknowns from three givens. That is:

At any location on Earth, given:

1- $\lambda_E \equiv$ the latitude of Earth station

2- $\phi_E \equiv$ the longitude of Earth station

3- $\phi_{SS} \equiv$ the longitude of the satellite we calculate :

1- $d \equiv$ the distance from Earth Station to the satellite.

2- $El \equiv$ Elevation angle from the horizon at the Earth station.

3- $Az \equiv$ Azimuth angle from the North direction

Notes:

1- $\lambda_{ss} = 0$ since all GSO satellites are directly above the equator.

2- The $El$ & $Az$ angles are required to steer the main beam of the Earth station antenna

# Procedure;

Given ( $\lambda_E$, $\phi_E$, $\phi_{ss}$ )

1- $\cos b = \cos(\phi_E - \phi_{ss}) \cos \lambda_E$

2- $d = \sqrt{R_E^2 + a_{GSO}^2 - 2 R_E a_{GSO} \cos b}$

3- $\cos(El) = \dfrac{a_{GSO}}{d} \sin b$

4- $\sin A = \dfrac{\sin|\phi_E - \phi_{ss}|}{\sin b}$

5- $$A_z = \begin{cases} A, & \lambda_E < 0 \;\&\; \phi_E < \phi_{ss} \\ -A, & \lambda_E < 0 \;\&\; \phi_E > \phi_{ss} \\ 180 - A, & \lambda_E > 0 \;\&\; \phi_E < \phi_{ss} \\ -(180 - A), & \lambda_E > 0 \;\&\; \phi_E > \phi_{ss} \end{cases}$$

Special Cases:

(1) $\lambda_E = 0$ [Equitorial Earth station]

$$\cos b = \cos(\phi_E - \phi_{ss})$$

$$\Rightarrow \quad b = \phi_E - \phi_{ss}$$

$$d = \sqrt{R_E^2 + a_{GSO}^2 - 2R_E\, a_{GSO} \cos(\phi_E - \phi_{ss})}$$

$$\cos(El) = \frac{a_{GSO}}{d} \sin(\phi_E - \phi_{ss})$$

$$\sin A = \frac{\sin|\phi_E - \phi_{ss}|}{\sin(\phi_E - \phi_{ss})}$$

$$\Rightarrow \quad A_z = \begin{cases} +90^\circ, & \phi_E < \phi_{ss} \\ -90^\circ, & \phi_E > \phi_{ss} \end{cases}$$

(2) $\lambda_E = 0$ & $\phi_E = \phi_{SS}$

$$b = 0$$

$$d = a_{GSO} - R_E$$

$$El = 90^\circ$$

$A_z$ becomes meaningless because the satellite is directly overhead.

(3) $\phi_E = \phi_{SS}$ & $\lambda_E \neq 0$

$$\cos b = \cos \lambda_E$$

$$\Rightarrow b = \lambda_E$$

$$d = \sqrt{R_E^2 + a_{GSO}^2 - 2 R_E a_{GSO} \cos \lambda_E}$$

$$\cos(El) = \frac{a_{GSO}}{d} \sin \lambda_E$$

$$A_z = \begin{cases} 0^\circ, & \lambda_E < 0 \\ 180^\circ, & \lambda_E > 0 \end{cases}$$

## 3.2. Polar Mount Antenna:

Many commercial antennas can be rotated on one axis only.

In this case, we adjust El to be $El_0$ where

$$\cos(El_0) = \frac{a_{GSO}}{d} \sin \lambda_E$$

The Elevation of the antenna is kept constant at this angle and then azimuth rotation is calculated according to the same procedure as before

where $\sin(A) = \frac{\sin|\phi_E - \phi_{ss}|}{\sin b}$,

& $b = \cos^{-1}\{\cos(\phi_E - \phi_{ss}) \cos \lambda_E\}$

$A_z$ is calculated from $A$ as before.

## 3.3. Limits of Visibility:

At any location, the elevation angle of the antenna must be greater than zero (above the horizon).

In this case, there is a limit on the longitude of the satellite to be visible from a given Earth station.

At the equator ($\lambda_E = 0°$), this limit is given by:

$$\cos(\phi_E - \phi_{SS}) \geq \frac{R_E}{a_{GSO}}$$

{ Note: The equality gives $El = 0°$ }

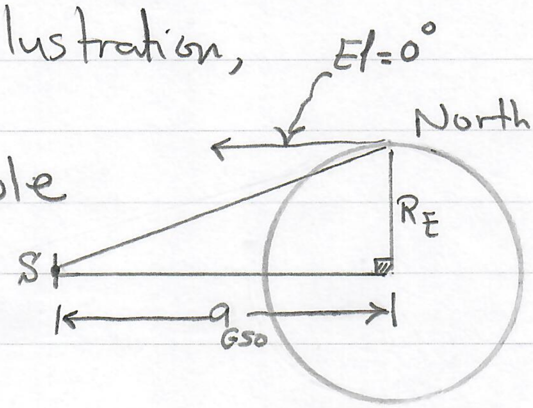
$$\Rightarrow \quad |\phi_E - \phi_{SS}|_{max} \cong 81°$$

In practice, this limit is even more restrictive.

This is because the minimum elevation angle should be higher than 0°. This is to avoid obstruction by surrounding objects and also to avoid ground reflections.

There are some places where the geostationary orbit cannot be seen at all.

As seen in this illustration, the north (and south) pole of Earth cannot see the GSO. This is because the required Elevation is actually less than 0°




The look direction whose elevation angle is zero goes parallel to the equator and never meets the GSO.

Thus, the limit on $|\phi_E - \phi_{ss}|$ is maximum (wide view) at the equator then it keeps decreasing as the $|\lambda_E|$ keeps increasing until it reaches zero at a certain $\lambda_E$, before even we reach the poles of the Earth.

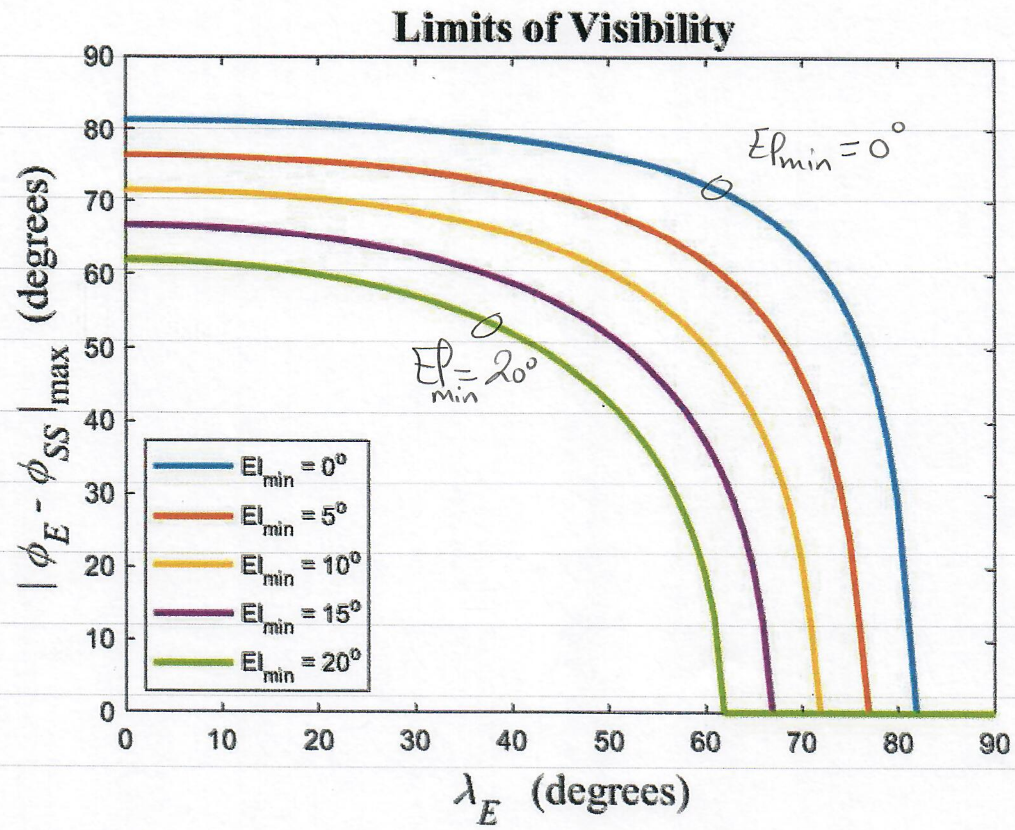
In other words for $El \geq 0°$, $\{$i.e. $El_{min} = 0°\}$

$$|\phi_E - \phi_{SS}|_{max} = \begin{cases} 81° & \text{at } \lambda_E = 0° \\ 0° & \text{at } |\lambda_E| = 81° \end{cases}$$

Due to practical reasons as mentioned above $El_{min}$ is set at a value higher than $0°$. This makes the value of $81°$ mentioned in the above equation gets even lower.

This effect is illustrated in the figure below

[A spherical Earth with $R = 6371$ km is assumed]




$$\cos|\phi_E - \phi_{ss}|_{max} = \frac{\cos b}{\cos \lambda_E}$$

where $b$ is taken here as:

$$b = 90° - El°_{min} - \sin^{-1}\left(\frac{R_E}{a_{GSO}} \sin(90° + El°_{min})\right)$$

{Note: the proof is not required from you. Only understand the behaviour of $|\phi_E - \phi_{ss}|_{max}$ with $\lambda_E$ }

This problem of limited visibility of the GSO affected Russia the most since its territory is mostly on the extreme north of the planet.

To solve this problem, Monliya orbits were utilized.

Molniya orbits are inclined elliptical orbits with the following properties

Eccentricity = 0.74 (highly elliptical)

Semi-major axis = 26,600 km

Inclination ≅ 65°

Period ≅ 12 hours {two revolutions per day}

Argument of perigee = 270°

At apogee, a Molniya satellite reaches the geostationary orbit, while at perigee it is as low as ~300 km above sealevel.

According to keplere's second law, the Molniya satellites will be much slower at apogee than they are at perigee.

About three Molniya satellits in the same orbit can cover the northern hemisphere throughout the day without the need for any change in the look direction of the Earth station antenna.

Use the tools learned in chapter 2 to plot a visualization for the Molniya orbit.