Chapter (5)

Polarization

5.1. Polarization of Satellite signals.

At the satellite in orbit, there are only to directions to define the wave polarization. The horizontal polarization is sent with an electric field vector parallel to the equator. The vertical polarization is sent with an electric field vector parallel to the North-South poles line. These directions, however, are not the same at the Earth Station.

At any Earth Station, the electric field vector must be perpendicular to the direction of propagation, "$\bar{k}$". Thus, the electric field vector lies in a plane perpendicular to $\bar{k}$. In this plane, we define a vector reference "$\bar{f}$"

to measure the angle of polarization with respect to it. We define a set of coordinates where the origin is at the center of the Earth. The z-axis is the North-south poles line and the satellite lies on the x-axis.

At an Earth station with position vector $\bar{R}$ given by: $\bar{R} = \hat{a}_x R_x + \hat{a}_y R_y + \hat{a}_z R_z$

$$= \begin{bmatrix} \hat{a}_x \, R_E \cos\lambda_E \cos(\phi_E - \phi_{ss}) \\ \hat{a}_y \, R_E \cos\lambda_E \sin(\phi_E - \phi_{ss}) \\ \hat{a}_z \, R_E \sin\lambda_E \end{bmatrix}$$

the direction of propagation $\bar{K}$ is given by:

$$\bar{K} = \bar{R} - \hat{a}_x a_{GSO} = \begin{bmatrix} \hat{a}_x K_x \\ \hat{a}_y K_y \\ \hat{a}_z K_z \end{bmatrix} = \begin{bmatrix} \hat{a}_x R_x - a_{GSO} \\ \hat{a}_y \; R_y \\ \hat{a}_z \; R_z \end{bmatrix}$$

The reference vector "$\bar{f}$" is given by:

$$\bar{f} = \bar{R} \times \bar{K} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ R_x & R_y & R_z \\ K_x & K_y & K_z \end{vmatrix}$$

$$= \begin{bmatrix} \hat{a}_x (R_y K_z - R_z K_y) \\ \hat{a}_y (R_z K_x - R_x K_z) \\ \hat{a}_z (R_x K_y - R_y K_x) \end{bmatrix}$$

Now, we need to know $\bar{p}$ which is the direction of the electric field vector at the earth station with respect to $\bar{f}$.

Without the effect of Earth's atmosphere we can say $\bar{P} \parallel \bar{h}$ where $\bar{h}$ depends on $\bar{g}$

and $\bar{g} = \bar{K} \times \bar{e}$

where $\bar{e}$ is the satellite signal polarization.

$\bar{e} = \hat{a}_y$ for horizontal polarization

& $\bar{e} = \hat{a}_z$ for vertical polarization

$$\Rightarrow \quad \bar{g} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ k_x & k_y & k_z \\ 0 & 1 & 0 \end{vmatrix}$$ for horizontal poli

$$\& \quad \bar{g} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ k_x & k_y & k_z \\ 0 & 0 & 1 \end{vmatrix}$$ for vertical Pol.

Then we define $\bar{h}$ as

$$\bar{h} = \bar{g} \times \bar{k}$$

& $\bar{p}$ is a unit vector in direction of $\bar{h}$.

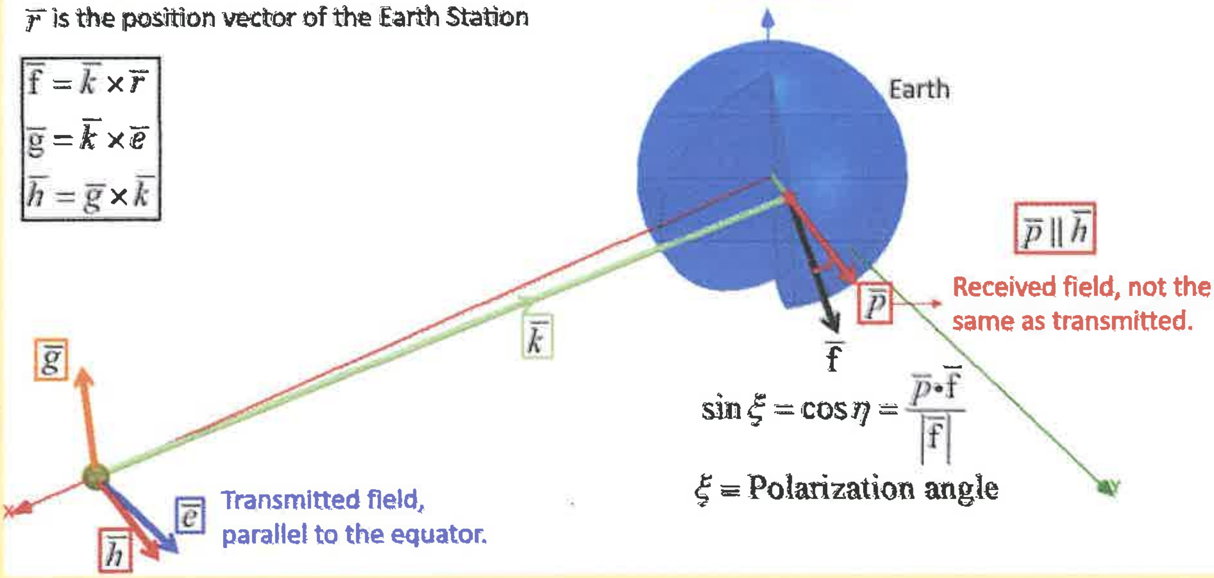
The polarization angle $\xi = \sin^{-1}\left(\frac{\bar{p}\cdot\bar{f}}{|\bar{f}|}\right)$

or easier we take $\eta = \cos^{-1}\left(\frac{\bar{p}\cdot\bar{f}}{|\bar{f}|}\right)$

When Satellite sends Horizontal Polarization

$\overline{f}$ is the polarization reference at the Earth Station
$\overline{r}$ is the position vector of the Earth Station

$$\overline{f} = \overline{k} \times \overline{r}$$
$$\overline{g} = \overline{k} \times \overline{e}$$
$$\overline{h} = \overline{g} \times \overline{k}$$



$$\sin\xi = \cos\eta = \frac{\overline{p}\cdot\overline{f}}{|\overline{f}|}$$

ξ = Polarization angle

Example: $\lambda_E = 0^\circ$, $\phi_E = \phi_{ss} + 30^\circ$

Find η & ξ

Sol.

$$\overline{R} = \begin{bmatrix} \hat{a}_x & R_E \cos\lambda_E \cos(\phi_E - \phi_{ss}) \\ \hat{a}_y & R_E \cos\lambda_E \sin(\phi_E - \phi_{ss}) \\ \hat{a}_z & R_E \sin\lambda_E \end{bmatrix} = \begin{bmatrix} \hat{a}_x & R_E \cos 30^\circ \\ \hat{a}_y & R_E \sin 30^\circ \\ & 0 \end{bmatrix}$$

$$\overline{K} = \begin{bmatrix} \hat{a}_x & R_E \cos 30^\circ - a_{GSO} \\ \hat{a}_y & R_E \sin 30^\circ \\ & 0 \end{bmatrix}$$

$R_E = 6371$ km
$a_{GSO} = 42{,}164$ km

$$\bar{f} = \bar{R} \times \bar{K} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ R_x & R_y & 0 \\ K_x & K_y & 0 \end{vmatrix}$$

$$= \hat{a}_z \left(R_x K_y - R_y K_x \right)$$

$$\bar{g} = \bar{K} \times \bar{e} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ K_x & K_y & 0 \\ 0 & 1 & 0 \end{vmatrix}$$

$$= \hat{a}_z K_x$$

$$\bar{h} = \bar{g} \times \bar{K} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & 0 & K_x \\ K_x & K_y & 0 \end{vmatrix}$$

$$= \hat{a}_x (-K_x K_y) + \hat{a}_y (K_x)^2$$

$\because \bar{P} \parallel \bar{h}$

$$\therefore \bar{P} \cdot \bar{f} = \frac{\left[\hat{a}_x (-K_x K_y) + \hat{a}_y (K_x)^2 \right] \cdot \hat{a}_z \left[K_x R_y - R_y K_x \right]}{|\bar{h}|\ |\bar{f}|}$$

$$= 0 \Rightarrow \boxed{\eta = 90^\circ \ \& \ \xi = 0^\circ}$$

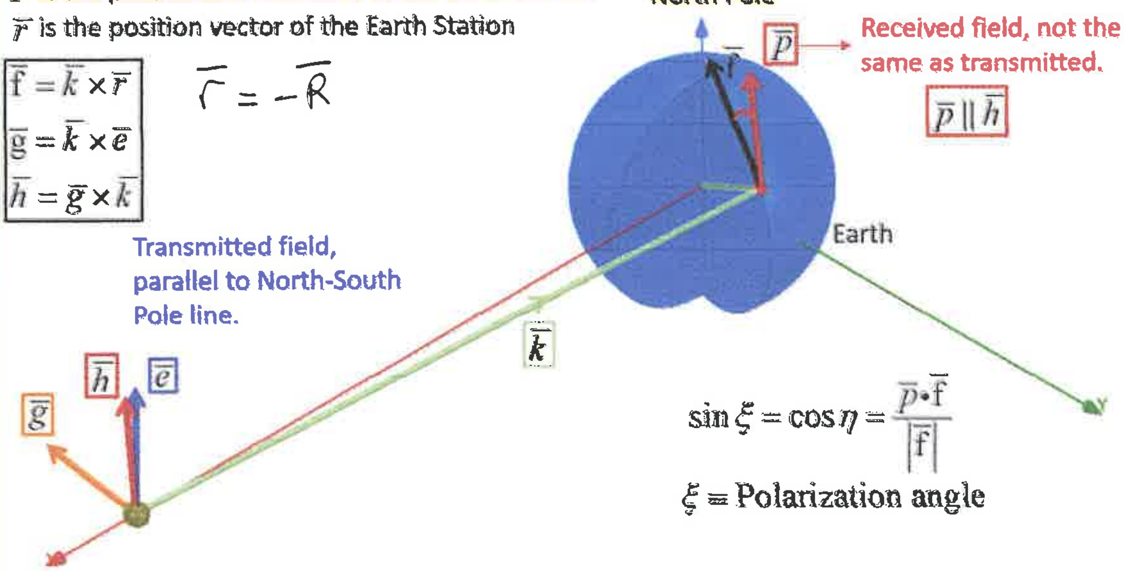
When Satellite sends Vertical Polarization

$\overline{f}$ is the polarization reference at the Earth Station

$\overline{r}$ is the position vector of the Earth Station

$$\boxed{\begin{aligned}\overline{f} &= \overline{k}\times\overline{r}\\ \overline{g} &= \overline{k}\times\overline{e}\\ \overline{h} &= \overline{g}\times\overline{k}\end{aligned}}$$

$\overline{r} = -\overline{R}$



$$\sin\xi = \cos\eta = \frac{\overline{p}\cdot\overline{f}}{|\overline{f}|}$$

ξ = Polarization angle

Example: $\lambda_E = 0^\circ$ & $\phi_E = \phi_{ss} + 30^\circ$

find ξ & η

Sol.

$$\overline{R} = \begin{bmatrix} \hat{a}_x\, R_E \cos\lambda_E \cos(\phi_E - \phi_{ss}) \\ \hat{a}_y\, R_E \cos\lambda_E \sin(\phi_E - \phi_{ss}) \\ \hat{a}_z\, R_E \sin\lambda_E \end{bmatrix} = \begin{bmatrix} \hat{a}_x\, R_E \cos 30^\circ \\ \hat{a}_y\, R_E \sin 30^\circ \\ \hat{a}_z\ \ 0 \end{bmatrix}$$

$$\overline{k} = \begin{bmatrix} \hat{a}_x\, R_E \cos 30^\circ - a_{GSO} \\ \hat{a}_y\, R_E \sin 30^\circ \\ 0 \end{bmatrix}$$

$$\bar{f} = \bar{R} \times \bar{k}$$

$$= \hat{a}_z (R_x k_y - R_y k_x)$$

$$\bar{g} = \bar{k} \times \bar{e} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ k_x & k_y & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \hat{a}_x k_y - \hat{a}_y k_x$$

$$\bar{h} = \bar{g} \times \bar{k} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ k_y & -k_x & 0 \\ k_x & k_y & 0 \end{vmatrix}$$

$$= \hat{a}_z (k_y^2 + k_x^2)$$

$$\therefore \bar{p} \parallel \bar{h}$$

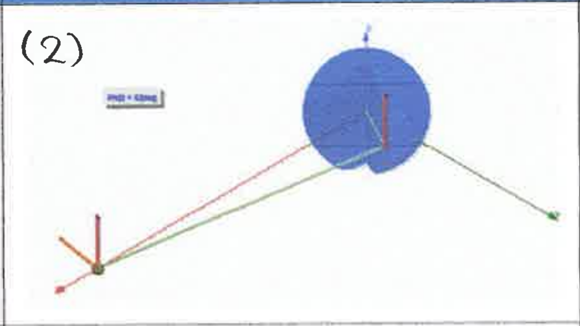
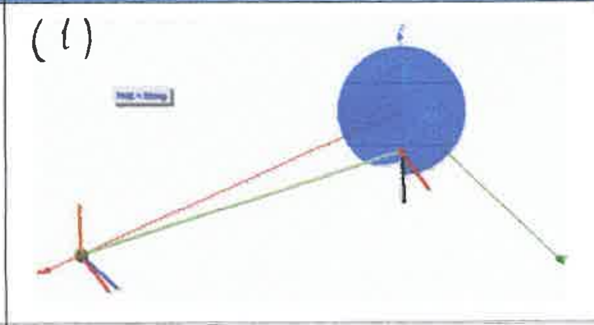
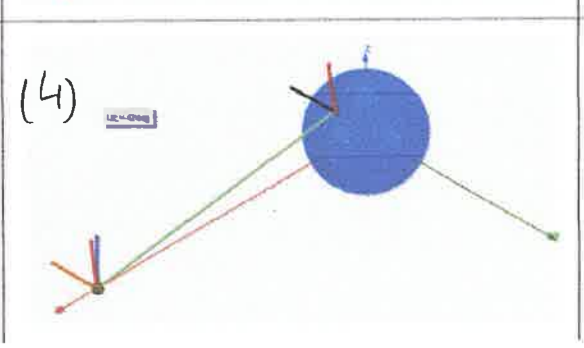
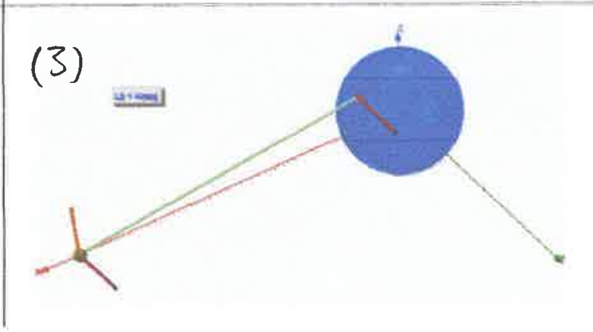
$$\therefore \bar{p} = \hat{a}_z$$

$$\Rightarrow \xi = \sin^{-1} \frac{\bar{p} \cdot \bar{f}}{|\bar{f}|} = \sin^{-1} (\hat{a}_z \cdot \hat{a}_z) = \sin^{-1} 1$$

$$= 90^\circ$$

$$\& \ \boxed{\eta = 90 - \xi = 0^\circ}$$

Special Cases

	Vertical Polarization	Horizontal Polarization
$\lambda_E = 0^\circ$	(2) 	(1) 
$\varphi_B = 0^\circ$	(4) 	(3) 

There are four special cases:

(1) $\lambda_E = 0$ & $\phi_E - \phi_{SS} \neq 0$ & $\bar{e} = \hat{a}_y$ (H-pol)

(2) $\lambda_E = 0$ & $\phi_E - \phi_{SS} \neq 0$ & $\bar{e} = \hat{a}_z$ (V-pol.)

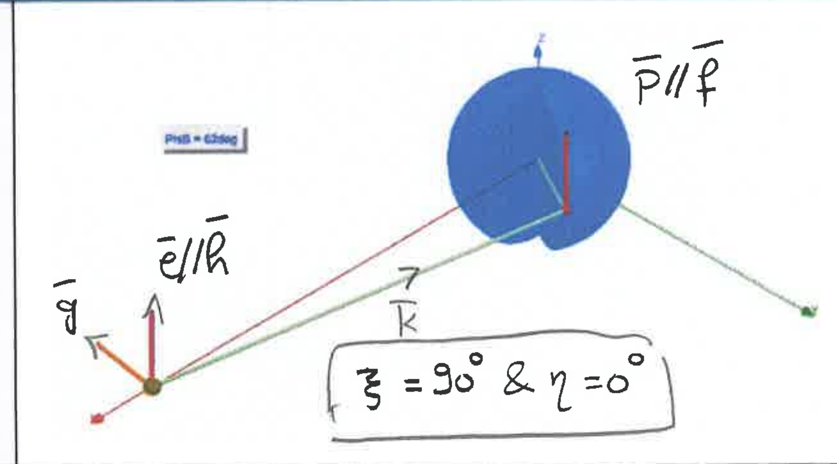
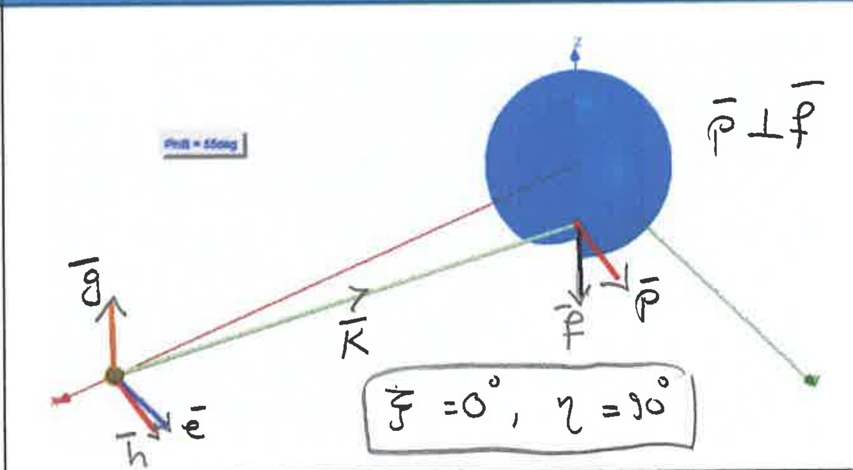
(3) $\lambda_E \neq 0$ & $\phi_E - \phi_{SS} = 0$ & $\bar{e} = \hat{a}_y$ (H-pol.)

(4) $\lambda_E \neq 0$ & $\phi_E - \phi_{SS} = 0$ & $\bar{e} = \hat{a}_z$ (V-pol.)

* Try to think about the special case where

$\lambda_E = 0$ & $\phi_E - \phi_{SS} = 0$

Special Cases

<table>
<tr><th></th><th>Vertical Polarization</th><th>Horizontal Polarization</th></tr>
<tr><td>$\lambda_E = 0^\circ$</td><td>
</td><td>
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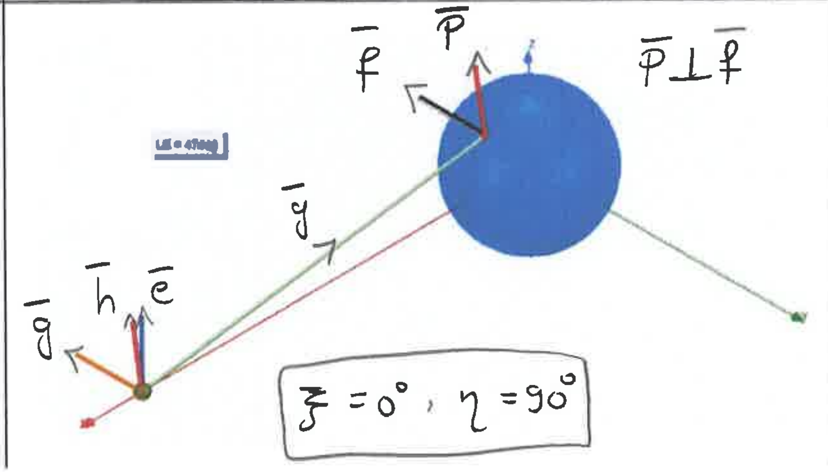
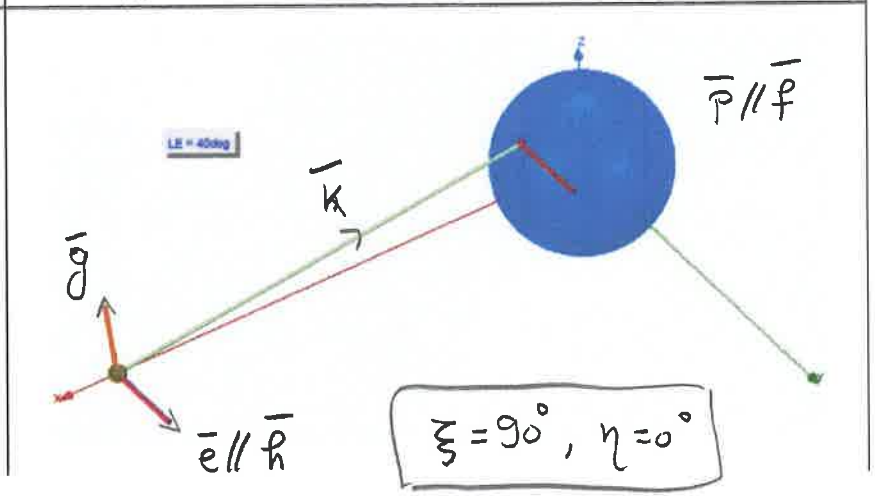
<tr><td>$\varphi_B = 0^\circ$</td><td>
</td><td>
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